19. If $F=(2xy+z^3)\vec{i}+x^2\vec{j}+3xz^2\vec{k}$ find $\int_C F.dr$ where C is any path joining (1, –2, 1) to (3, 1, 4).

20. Expand $f(x)=x^2+x(-\pi<x<\pi)$ in a Fourier Series.

APRIL/MAY 2023

FAEL22/CAEL22 — BASIC MATHEMATICS – II

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Write the necessary condition for the function to be maximum.

2. Find $\frac{\partial(u,v)}{\partial(x,y)}$ if $u=x^2, v=y^2$

3. Solve $(D^4-1)y=0$

4. Find the particular integral of $(D^2+D+1)y=\cos x$.

5. Define singular integral.

6. Eliminate a and b from $z=(x^2+a)(y^2+b)$

7. Prove that $\nabla.r=3$ and $\nabla\times r=0$

8. State Stoke's theorem.

9. Evaluate $\int_0^{\pi/2} \sin^6 x dx$

10. Write the formula for Fourier Constants for $f(x)$ in the interval $(-\pi, \pi)$

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Find y_n when $y = \frac{x^2}{(x-1)^2(x+2)}$

Or

(b) If $y = (\sin^{-1} x)^2$. Prove that $(1-x^2)y_2 - xy_1 - 2 = 0$

12. (a) Solve $(D^2 - 3D + 2)y = \cosh x$

Or

(b) Solve $(D^3 - D^2 - 6D)y = 1 + x^2$

13. (a) Solve $p^2 + q^2 = npq$

Or

(b) Solve $p + q = x + y$

14. (a) Compute the divergence and curl of the vector $F = xyz\vec{i} + 3x^2y\vec{j} + (xz^2 - y^2z)\vec{k}$ at (1, 2, –1).

Or

(b) Given the vector field $F = xz\vec{i} + yz\vec{j} + z^2k$, evaluate $\int_C F.dr$ from the point (0, 0, 0) to (1, 1, 1), where C is the curve $x = t$, $y = t^2$, $z = t^3$.

15. (a) Evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$

Or

(b) Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Show that the least value of $a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x$ is $(a+b)^2$

17. Solve $(D^2 + 4D + 5)y = e^x + \cos 2x$

18. Solve $Z = p^x + qy + \sqrt{(1 + p^2 + q^2)}$